Loan Sales and the Cost of Bank Capital

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ABSTRACT

This paper considers a model where banks may improve the returns on loans by monitoring borrowers. Bank regulation, together with competitive deposit and equity financing, can give banks an incentive to sell loans, but the extent of their loan selling is limited by a moral-hazard problem. A solution is given for the optimal design of the bank-loan buyer contract that alleviates this moral-hazard problem. An explanation is also given as to why some banks might buy loans and why loan sales volume has recently increased.

Banks’ practice of making loans and then selling them to other institutions and individuals has grown in popularity. An important example in the development of loan selling has been the increase in banks’ mortgage loans that are insured and pooled under the authority of the Government National Mortgage Association (GNMA) and then sold to secondary market investors. Recently, however, there has been a dramatic rise in the volume of other types of loans being sold, especially by money-center banks. Portions of commercial loans originated by these larger banks are being sold to smaller banks, foreign banks, and other financial and nonfinancial institutions. In addition, banks’ car loans and credit card receivables have also been pooled and sold to institutions and individuals. 1 Selling loans that were once considered nonmarketable assets, a process that has been termed “asset securitization”, may be signalling the start of a fundamental change in the commercial banking business. The leading banks in loan-selling operations now view themselves more as originators and distributors of loans rather than as institutions holding loans as assets.

The potentially large impact of loan sales on the future of commercial banking naturally evokes the question of what incentives exist for banks to sell loans. In this paper, we show that loan sales allow some banks to finance loans less expensively than by traditional deposit or equity issue because bank funds received via loan sales can avoid costs associated with required reserves and

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required capital. However, it is worth noting that current research suggests that there may be additional reasons why funding through loan sales can be less expensive than bank deposits and equity. Greenbaum and Thakor [14] show that signalling information regarding loan quality may be enhanced when loans are sold rather than funded by deposits. Another recent paper by James [17] demonstrates that loan sales can provide lower cost financing for bank equity-holders and enable the bank to avoid a possible underinvestment problem when it has risky debt outstanding. In addition, Flannery [11] shows how current bank-examination procedures may induce banks to hold only certain risk classes of loans while profitably selling the rest.

Our paper goes on to demonstrate that the extent of banks’ loan selling is limited by a moral-hazard problem that arises from the diminished incentive by banks to efficiently monitor and service loans after they have been sold. Banks can help alleviate this problem by optimally designing their contracts with loan buyers. By offering loan buyers an incentive-efficient loan sales contract, a bank’s loan sales volume, and hence its profitability, can be maximized.

The plan of the paper is as follows. In Section I, we present a simple state-preference model of the banking firm. The bank chooses its optimal quantity of loans to originate along with its levels of monitoring borrowers. The bank also decides whether to finance its asset holdings by issuing deposits or equity. We examine optimal portfolio and capital-structure decisions, first when loan sales are prohibited and second when they are allowed. In order to determine a bank’s equilibrium quantity of loan sales, the optimal bank-loan buyer contract is studied next. In Section II, we analyze the optimal design of this contract when the level of loan monitoring by banks is unobservable and agents are risk neutral. Contracts are considered where the loan buyer has no recourse to the bank for losses and also where recourse is permitted. Section III generalizes the model to allow banks market power in deposit financing. We show that equilibria exist where some banks will choose to sell loans while other banks may choose to purchase loans. This analysis leads to an explanation for why the aggregate volume of loan selling has recently increased. A conclusion follows in Section IV.

I. A Model of the Banking Firm

This model concentrates on the loan-making activities of banks. Originating loans, as well as possibly holding marketable securities, is what we will define as the “portfolio” services provided by banks—the process of channeling funds between savers and borrowers. It is assumed that banks can expend real resources in gathering information on loan applicants and monitoring loans so as to improve the return (quality) on these loans. The information-gathering and monitoring functions of banks have been stressed in the papers by Campbell and Kracaw [4] and Diamond [6]. Other services provided by banks, particularly transactions services such as check clearing and deposit-to-currency convertibility, are ignored in this paper. As in Black [2], Fama [8], and Fischer [10], it is assumed that no necessary connection exists between the portfolio and transactions services of
banks, and hence they can be separated into different “departments” of the banking organization.²

A. Equilibrium with No Loan Sales

Consider a one-period state-preference model where banks choose investments in loans or marketable securities. Let \( N \) denote a bank’s total amount of funds initially invested in these assets. We assume that a technology exists where a bank can improve the (uncertain) return on its loans by expending resources on gathering information and monitoring its borrowers.

(A1) Banks can make unit investments such that one dollar lent to borrower \( i \) at the beginning of the period entitles the bank to an end-of-period cash flow equaling \( x_i(s, a_i) \), where \( s \) indexes the state of nature at the end of the period and \( a_i \) is the level of “monitoring” chosen by the bank at the beginning of the period.³ Assuming a possibly infinite number of states, \( s \) can be designated as \( s \in [0, 1] \), i.e., real numbers on the unit closed interval. \( x_i(s, a_i) \) is a concave function of \( a_i \).

(A2) Banks produce monitoring services, \( a_i \), via a constant-returns-to-scale technology so that their cost function is \( c(a_i) = ca_i \), where \( c \) is a positive constant.

Assumptions (A1) and (A2) describe the bank’s investment opportunities. For investments such as loans to small firms and consumers, it is reasonable to believe that \( \frac{\partial x_i(s, a_i)}{\partial a_i} \geq 0 \); i.e., monitoring services, produced from inputs such as the labor of loan officers and assessors and computer hardware and software, can improve the bank’s return on these loans. However, in the case of loans to well-known investment-grade corporations and investments in money-market assets such as Treasury bills, it is more reasonable to assume that bank monitoring would have no effect on these marketable assets’ cash flows, so that \( \frac{\partial x_i(s, a_i)}{\partial a_i} = 0 \) for these assets.

A bank can finance its investments by issuing deposits or equity. Let \( D \) denote the bank’s total level of deposit financing, and let \( E \) equal the amount of equity funds raised by the bank.

(A3) Banks are price takers in both deposit (debt) and equity financing markets. Assuming complete markets, let \( p^d(s) \) denote the equilibrium price (density) paid by depositors (debtholders) for a security having no agency costs that pays one dollar in state \( s \) at the end of the period, where the return on this security is treated as debt for personal tax purposes.

² Extending the model to allow for a transactions technology and for transactions deposit accounts produces no substantive changes in the model’s results concerning loan sales.
³ While this effort expended by the bank for a given loan, \( a_i \), is referred to as the bank’s level of monitoring activities, this effort could also be interpreted to include the bank’s information-gathering and credit-checking activities necessary to select a better quality loan from a pool of applicants. Therefore, our reference to a moral-hazard problem of inefficient monitoring caused by the unobservability of the bank’s effort by loan buyers, which is explained in Section I, Subsection B, and treated in Section II, could be interpreted to refer also to an adverse-selection problem of inefficient information gathering by the bank.
Similarly, $p^e(s)$ denotes the equilibrium price density paid by equity-holders for a security paying one dollar in state $s$ and with a return taxed as equity.

(A4) Banks are subject to a corporate income tax, with proportional tax rate equal to $t$. They are also required to hold non-interest-bearing reserves on deposits where $\rho$ is the required reserve/deposit ratio.

Assumption (A3) assumes perfect competition in financing bank investments. However, we will generalize the analysis to consider imperfect competition in deposit markets later in Section III. Since this paper disregards transactions services of banks, one should think of banks' deposits as being in non-transactions accounts, such as money-market deposit accounts, certificates of deposit, or other "purchased funds," that thus can be regarded simply as (short-term) debt instruments. In practice, non-transactions deposits have either zero or small reserve requirements. For example, money-market deposit accounts have no required reserves, while most certificates of deposit have a three percent requirement. Whether $\rho$ is zero or small will not be critical in terms of our qualitative results.

Using the price of a security that pays one dollar in all states, $s \in [0, 1]$, we can define $r_d$ and $r_e$ as the certainty-equivalent required returns to debtholders and equityholders, respectively:

$$\frac{1}{1 + r_i} = \int_0^1 p^i(s) \, ds, \quad i = d, e. \quad (1)$$

Bank deposits are assumed to be insured by a government regulator who also imposes an initial capital requirement on the bank.

(A5) Deposits are insured by a government agency that charges the bank a (risk-sensitive) premium covering the value of the agency's end-of-period deposit guarantee. However, the insurer limits banks' selection of a deposit (debt)-equity ratio to a maximum of $\xi$. Because of the insurer's special legal and regulatory authority over banks, we assume that it can costlessly monitor a bank's activities such that the bank's investment and financial decisions are fixed after its insurance premium is set.

The assumption that deposit insurance is fairly priced is made for a number of reasons. First, while distortions created by mispriced deposit insurance have been studied in a number of papers, whether deposit insurance is actually over- or underpriced for most banks is an unresolved issue.\(^4\) Second, less-than-substantial mispricing does not lead to qualitatively different results but adds more clutter to the analysis. While risk-insensitive deposit insurance may influence a bank's desire to sell loans, we show that this is not a necessary condition for a bank loan-selling incentive.\(^5\) Third, the fair-pricing assumption may be

\(^4\) In a similar state-preference framework, Dothan and Williams [7] analyze distortions arising from the mispricing of deposit insurance. Pennacchi [29] examines whether deposit insurance provided by the FDIC is generally over- or underpriced.

\(^5\) Because of the presence of a capital-adequacy constraint in our model, unlike Dothan and Williams [7], the incentive for banks to pursue greater risk, normally associated with fixed-rate or
justified by appealing to the analysis of Buser, Chen, and Kane [3], who suggest that a riskier bank faces a greater regulatory cost imposed on it by the FDIC. From the bank's point of view, this cost serves the same function as a risk-sensitive (implicit) deposit insurance premium.

The assumption of costless monitoring by the insuring agency is meant to capture one effect of the existence of deposit insurance and bank regulation. Deposit insurance can be viewed as an institutional structure that results in the agency costs associated with uninsured deposits being reduced. Merton [22, p. 3] states, "Hence, for the small depositor particularly, there are large information and surveillance costs to be saved if the institutional structure of the bank were such that the safety of the deposits was assured . . . ." A deposit insurer such as the FDIC has the regulatory authority to audit banks at will, require capital, issue cease-and-desist orders, and close banks, which will likely give it a monitoring cost substantially lower than that of uninsured debtholders or depositors who may be subject to free-rider problems. The imposition of a capital require-
ment also makes the zero-monitoring-cost assumption more reasonable since a capital constraint can be used to limit banks to leverage ratios where monitoring costs are sufficiently small.\footnote{The assumption of fair deposit insurance induces the bank to maximize the after-tax value of the firm. The fair insurance premium simply equals the value of the insurer's end-of-period liability since the insurer's monitoring costs are assumed to be zero. If monitoring costs were positive, they would need to be included in the calculation of a fair premium. However, sufficiently small costs would not overturn our results.}

The Appendix to this paper shows that, if the bank's objective is to maximize the after-tax gain to shareholders' equity, assumptions (A1) to (A5) lead to its objective function having the following form:  

\[
\max_{\{N,(a_i,D,E)\}} \left\{ \sum_{i=1}^{N} \left[ (1 + r_e) \int_0^1 p^e(s) x_i(s,a_i) \, ds - 1 - ca_i \right] - r_d D \right\} (1 - t) - r_e E, \tag{2}
\]

subject to the constraints:

\[
N + \rho D \leq D + E \quad \text{(financing constraint),} \tag{3}
\]

\[
D \leq \xi E \quad \text{(capital constraint).} \tag{4}
\]

In expression (2), we choose the bank's assets \(i, i = 1, \ldots, N\), to be ordered from the highest valued asset to the lowest.\footnote{This is assuming that each \(a_i\) is chosen optimally according to condition (6).} Thus, loan or security \(i = N\) is the "marginal" investment made by the bank, i.e., the investment that the bank is (approximately) indifferent to making.

Differentiating with respect to each of the bank's choice variables, this leads
to the first-order Kuhn-Tucker conditions:\footnote{1}

\[
\left\{(1 + r_e) \int_0^1 p^e(s) x_N(s, a_N) \, ds - 1 - c a_N \right\} (1 - t) - \lambda_1 \right\} N = 0, \tag{5}
\]

\[
\left\{(1 + r_e) \int_0^1 p^e(s) \frac{\partial x_i}{\partial a_i} (s, a_i) \, ds - c \right\} a_i = 0, \tag{6}
\]

\[
\{-r_d (1 - t) + \lambda_1 (1 - \rho) - \lambda_3 \} D = 0, \tag{7}
\]

\[
\{-r_e + \lambda_1 + \lambda_3 \} E = 0, \tag{8}
\]

where \( \lambda_1 \) and \( \lambda_2 \) are the multipliers associated with constraints (3) and (4), respectively, and the expressions in brackets are all nonpositive.

The bank's optimal choice of debt versus equity financing is given by conditions (7) and (8). The capital constraint of the bank will be binding; i.e., it will choose to be at its maximum debt/equity ratio, \( \zeta \), if

\[
\frac{r_d (1 - t)}{(1 - \rho)} < r_e, \tag{9}
\]

while reversal of this inequality implies that an all-equity capital structure is optimal.

A net tax advantage to debt financing (condition (9) for sufficiently small \( \rho \)) is supported by empirical evidence.\footnote{2} Moreover, in a theoretical model where the typical firm experiences agency costs of debt issue, Barnea, Haugen, and Senbet \cite{1} derive condition (9) (for \( \rho = 0 \)) as a general-equilibrium result. In that model, a firm chooses its debt/equity ratio where its rising marginal agency cost of debt issue equals the marginal net tax benefit to debt financing. It then follows (see Orgler and Taggart \cite{25, p. 218}) that, if insured banks have zero or negligible agency costs of deposit issue, they will choose to be at their maximum allowed debt/equity ratio. By assuming fairly priced deposit insurance with costless monitoring by the insurer, which is equivalent to assuming zero agency costs of debt issue, the relative tax advantages of debt are not fully realized at the binding debt/equity ratio.

Thus, with a binding capital constraint, the bank's choice of investments can now be determined. From the first-order conditions (5), (6), (7), and (8), we have

\( \footnote{1}{\text{Condition (5) is obtained by differentiating with respect to } N \text{ but ignoring the integer constraint on loans.}} \)

\( \footnote{2}{\text{Empirical evidence by Gordon and Malkiel \cite{12} and Skelton \cite{32} generally supports the hypothesis of a net tax advantage to debt. For example, Gordon and Malkiel find that (personal) tax-exempt yields are approximately seventy-five percent of taxable yields. If (1 - t) is approximately fifty-two to fifty-four percent during their sample period, this implies a net tax advantage to debt. Also, with current tax-law changes resulting in a thirty-four percent corporate tax rate, which exceeds the top twenty-eight percent marginal tax rate for high-income individuals, it can be argued that the model by Miller \cite{23}, in which firms are indifferent between debt and equity financing, cannot possibly hold. Casual evidence also suggests that bankers view equity as more expensive than debt or deposits since many bankers see increases in capital requirements as costly. For example, see The Wall Street Journal, December 2, 1985, "Capital-Ratio Rise Sours Bank Growth."}} \)
that the bank's optimal asset volume, \( N \), will satisfy

\[
(1 + r_e) \int_0^1 p^*(s) x_N(s, a_N^*) \, ds - 1 = ca_N^* + \left[ \frac{r_c}{1 - t} + \xi r_d \right] \left( 1 + \xi(1 - \rho) \right). \tag{10}
\]

The left-hand side of (10) is the value of contingent interest income received from the marginal one-dollar bank investment when the level of monitoring, \( a_N^* \), is optimally chosen according to condition (6). The right-hand side of (10) is simply the cost of making this marginal one-dollar investment. The first term on the right is the additional resource cost of monitoring the borrower, while the second term is the bank's weighted marginal cost of debt and equity issue.

Consider the implications of condition (10) for a bank's choice of marketable securities such as Treasury bills or similar money market instruments. Since \( \partial x_i(s, a_i) / \partial a_i = 0 \) for these securities, we know that \( a_N^* = 0 \). Further, if insured bank deposits, such as certificates of deposit, are competitively priced and have virtually the same risk, liquidity, and personal tax treatment as other money-market instruments, we can treat them as perfect substitutes, as does Fama [9]. In a competitive equilibrium, they must have the same yield, so that marginal revenue from money-market instruments, i.e., the left-hand side of (10), must equal \( r_d \). However, from (9), \( r_d \) is less than the marginal cost of bank financing, the right-hand side of (10), implying that, if bank deposits require a similar yield to money-market assets, it is unprofitable for banks to hold these assets solely for portfolio (non-transactions) purposes.\(^\text{10} \) Banks' added costs from required reserves and/or relatively expensive equity capital imply that, if their deposits are competitively priced, then their asset portfolios must yield more than just the return on marketable assets. Banks facing competitive financing must make other types of investments such as loans that require monitoring and credit checking—specialized activities not in direct competition with (unregulated) money market funds or individual investors. The next section shows how loan selling can be a profitable arrangement for banks that originate loans.

B. Equilibrium with Loan Sales

Consider a bank selling a claim on the return of a loan it originates. The interest on the loan buyer's funds used to buy this claim is assumed to be taxed in the same manner as interest on a standard (taxable) bond. While the optimal bank-loan buyer contract will be examined in more detail in the next section, here we simply assume that a share, \( b_i \), of the return from each loan \( i \) is sold to a loan buyer. Thus, if loan \( i \) requires one dollar of initial financing and returns \( x_i(s, a_i) \) dollars if state \( s \) occurs and the bank monitors at level \( a_i \), loan buyers

\(^\text{10} \) Fama [9] gives empirical evidence and a theoretical explanation similar to this paper's to show why certificates of deposit will have a yield nearly identical to similar money-market instruments, in spite of certificates of deposit required reserves. Because the "reserve tax" falls not on depositors but on the bank, Fama reaches the same conclusion as this paper, that banks issuing certificates of deposit will choose to "hold no open-market securities."
receive $b_i x_i(s, a_i)$ while the bank receives $(1 - b_i) x_i(s, a_i)$. The amount paid by loan buyers for a share, $b_i$, of loan $i$ is then

$$\bar{b}_i = b_i \int_0^1 p^d(s)x_i(s, a_i) \, ds. \quad (11)$$

Suppose that banks finance a proportion, $\bar{b}$, of each one-dollar loan through loan sales; i.e., $\bar{b}_i = \bar{b}$, $i = 1, \ldots, N$. Assuming fairly priced deposit insurance, the objective function for the bank is

$$\max_{[N, (a_i), D, E]} \left\{ \sum_{i=1}^N \left[ (1 + r_e) \int_0^1 p^e(s)(1 - b_i)x_i(s, a_i) \, ds - (1 - \bar{b}) - c a_i \right] - r_d D \right\} \times (1 - t) - r_e E, \quad (12)$$

subject to the same constraints (3) and (4) as before, except that now $(1 - \bar{b})N$ replaces $N$ in (3).

The bank's optimization problem now involves the following equilibrium condition regarding its choice of loans originated, $N$;

$$(1 + r_e) \int_0^1 p^e(s)x_N(s, a_N) \, ds - 1 = c a_N + r_f$$

$$- b_N \left[ (1 + r_f) \int_0^1 p^d(s)x_N(s, a_N) \, ds - (1 + r_e) \int_0^1 p^e(s)x_N(s, a_N) \, ds \right], \quad (13)$$

where $r_f = [r_e/(1 - t) + \xi r_m]/[1 + \xi(1 - \rho)]$ is the bank's weighted marginal cost of internal financing, equal to the bank's marginal cost of capital for the no-loan-sales case. Comparing the form of equation (13) with the analogous condition (10) for the no-loan-sales case, we see that the marginal cost of originating a loan, which is the right-hand side of (13), differs from that of (10) because of an additional final term. This term represents a possible savings to the bank in its marginal cost of capital due to raising funds via loan sales.

Under reasonable circumstances, this last term on the right-hand side of (13) will be less than zero. To see this, note from equation (1) that

$$\int_0^1 p^d(s) \, ds / \int_0^1 p^e(s) \, ds = (1 + r_e)/(1 + r_d). \quad (14)$$

This implies that the ratio of the "averages" of the primitive security prices for debt and equity equals the ratio of the certainty-equivalent rates of return on equity versus debt. An additional assumption that is sufficient, though not necessary, for loan sales to lower the marginal cost of capital would be that the debt/equity security price ratio be uniform across states:\textsuperscript{11}

$$p^d(s)/p^e(s) = (1 + r_e)/(1 + r_d) \quad \text{for all } s. \quad (15)$$

\textsuperscript{11} Litzenberger and Van Horne [19] show that, in a Miller [23] type of world with investors in different tax clienteles, the ratio of the rates of return for a primitive debt and equity security would equal the ratio of the complements of personal tax rates for equity and debt for the "marginal" investor. DeAngelo and Masulis [5] show that investor risk neutrality would imply this uniformity.
Employing this stronger assumption, the last term in equation (13) can be rewritten as

$$-b_N \left[ (1 + r_c) \int_0^1 p^*(s)x_N(s, a_N) \, ds \cdot \frac{(r_l - r_d)}{(1 + r_d)} \right] < 0$$  \hspace{1cm} (16)$$
since \( r_l > r_d \) from condition (9).

The loan-sales equilibrium condition (13) indicates that, if the price paid by loan buyers for their loan share sufficiently reflects the higher average relative price paid for debt securities over equity securities, then the bank can lower its cost of financing by selling loans. Note that this always holds in the certainty case, i.e., a single end-of-period state of the world. By selling loans, banks can raise funds at the same cost, \( r_d \), as deposits, but the funds acquired through loan sales do not appear as a larger level of deposits on the balance sheet of the bank. Therefore, the bank will not be required to issue more relatively expensive equity in order to stay within its capital adequacy constraint or be required to hold non-interest-paying reserves against these funds. Ceteris paribus, with a lower cost of capital, banks will choose to expand their loan-originating and monitoring activities, leading to a lower competitive interest rate on loans. From a macroeconomic perspective, an economy-wide increase in the proportion of loans sold would decrease the demand for high-powered money, tending to raise nominal measures of output.

Thus far, we have not addressed an important issue that affects the bank’s choice of monitoring when it decides to sell loans. If the bank’s monitoring levels, \( a_i, i = 1, \ldots, N \), are observable by loan buyers, enabling the bank to commit to given levels of monitoring, then loan sales can indeed lead to increased bank profits and loan originations.\(^{12}\) However, when loan monitoring is unobservable by loan buyers, a potential moral-hazard problem arises that can limit the proportion of loans sold. By selling a share of a loan, the bank’s incentive to monitor is diminished since its monitoring level will satisfy

$$(1 + r_c)(1 - b_l) \int_0^1 p^*(s) \frac{\partial x_i}{\partial a_i} (s, a_l) \, ds = c.$$  \hspace{1cm} (17)$$

With the marginal benefit from monitoring discounted by the factor \((1 - b_l)\), monitoring will be less than in the no-loan-sales case. Rational loan buyers will infer this diminished level of monitoring and hence expect a smaller state-contingent loan cash flow, \( x_i(s, a_l) \). Thus, they will pay the bank less per dollar of the loan the greater the total share of the loan sold.\(^{13}\) However, by optimally

\(^{12}\) To see this, suppose that banks can commit to the same levels of monitoring as in the no-loan-sales case, given by condition (6), i.e., \( a_i = a^* \). Then a comparison of the marginal loan equilibrium conditions (10) and (13) will imply a lower total marginal cost of an additional one-dollar loan, given the negativity of the last term on the right-hand side of (13). Furthermore, when monitoring is observable, if it is profitable to sell any share of a loan, it must be even more profitable to sell the entire loan.

\(^{13}\) The amount paid by loan buyers is still given by equation (11), but now loan buyers infer that the bank’s level of monitoring, \( a_i \), is that which satisfies equation (17), not equation (6). Since loan buyers can deduce this monitoring level with certainty, they know each \( x_i(s, a_l) \) with certainty, which justifies discounting these state-contingent cash flows by \( p^*(s) \).
structuring the bank-loan buyer contract, this moral-hazard problem can be reduced. Determining this optimal contract will help to explain the proportion of loans that can be sold.

II. Bank-Loan Buyer Contract Choice

A bank’s decision to originate and monitor a given loan can be made independently of the same decisions for other loans, as can be verified from the banks’ objective function (12) and conditions (13) and (17). Therefore, a separate objective function for each loan can be obtained. Taking condition (12) for the case of \( N = 1 \), multiplying by \( (1 - t) \), and substituting in the equilibrium condition of \( D = \xi E \), one obtains the individual loan objective function:

\[
(1 + r_x) \int_0^1 p^*(s)(1 - b)x(s, a) \, ds - ca - (1 + r_I)I,
\]

where \( I = (1 - \rho)D + E = 1 - \xi \) is the amount of internal financing used in originating the one-dollar loan.

In order to add more structure to our problem and keep the analysis tractable, the case in which security valuation by shareholders and loan buyers reflects risk neutrality is considered. In this case, investors will be concerned only with the expected return on their contingent claim to the loan. In addition, we make the following assumptions regarding the return distribution of the bank loan, the cost of monitoring, and observability.

(A1') The stochastic return on the loan, \( x \), has a distribution such that \( x \in [0, L] \), where \( L \) is the promised end-of-period payment on the loan. The bank can alter the loan’s return distribution by monitoring, so that the probability density function of the loan’s return has the form \( f(x, a) \). It is assumed, as in Hart and Holmström [15], that the loan’s distribution function, \( F(x, a) \), satisfies the convexity-of-distribution-function condition:

\[
F(x, \lambda a + (1 - \lambda)a') \leq \lambda F(x, a) + (1 - \lambda)F(x, a'), \quad \forall a, a'; \quad \lambda \in (0, 1).
\]

(A2') Let the bank’s cost of monitoring a loan be given by \( c(a) \), where \( c'(a) > 0 \) and \( c''(a) \geq 0 \).

(A3') Bank-loan monitoring is unobservable by loan buyers. However, they can observe the loan’s actual return, and, hence, their share of the loan’s return may be contingent on the loan’s actual return, i.e., \( b = b(x) \).

The bank’s problem of choosing the optimal loan sales contract and level of monitoring can then be written as

\[
\max_{b(x), I, \alpha} \int_0^L (1 - b(x))x \, dF(x, a) - c(a) - (1 + r_I)I,
\]
subject to
\[ \int_0^L b(x) x \, dF(x, a)/(1 + r_d) + I = 1 \text{ (financing constraint),} \quad (21) \]
\[ \int_0^L (1 - b(x)) x \, dF(x, a) - c(a) \geq \int_0^L (1 - b(x)) x \, dF(x, a') - c(a'), \quad \forall a' \neq a \text{ (incentive-compatibility constraint).} \quad (22) \]

However, if condition (19) holds, Hart and Holmström [15] show that the incentive-compatibility constraint, (22), can be converted into the more convenient form:
\[ \int_0^L (1 - b(x)) x \, dF_a(x, a) = c'(a). \quad (23) \]

We are now prepared to consider a variety of contractual arrangements between the bank and the loan buyer.

A. Loan Sales without Recourse

We will define a bank-loan buyer contract with no recourse as one in which the bank cannot pledge outside assets as a potential payment to the loan buyer. Only the proceeds of the loan return are permitted to be split between the bank and loan buyer. As will be discussed in greater detail in the next section, the Federal Reserve places restrictions on loans sold with recourse, and in practice the great majority of loan sales are made without recourse. Therefore, it is of interest to consider this case. The no-recourse restriction on the loan sales contract takes the form:
\[ b(x) \leq 1 \quad \text{for all } x. \quad (24) \]

Conditions (20), (21), (23), and (24) then characterize the bank’s problem of selecting the optimal no-recourse contract. Let \( \omega \) and \( \lambda \) be the Lagrange multipliers for the constraints (21) and (23), respectively, and let \( \mu(x) \) be the multipliers for the inequalities in (24).

The first-order conditions with respect to the bank’s choice of \( b(x), I, \) and \( a \) are

\[ \left\{ \frac{\omega}{1 + r_d} - 1 \right\} x f(x, a) - \lambda x f_a(x, a) - \mu(x) \right\} b(x) = 0, \quad (25) \]
\[ -(1 + r_I) + \omega |I = 0, \quad (26) \]
\[ \frac{\omega}{1 + r_d} \int_0^L b(x) x \, dF_a(x, a) + \lambda \left[ \int_0^L (1 - b(x)) x \, dF_a(x, a) - c''(a) \right] \right\} a = 0, \quad (27) \]

where the expressions in brackets must be nonpositive.

For the case where, in equilibrium, the entire loan is not financed through loan sales, i.e., \( I > 0 \), then from condition (26) we have that \( \omega = 1 + r_I \). Condition
(25) can then be written as

$$\theta x f(x, a) - \lambda x f_a(x, a) - \mu(x)b(x) = 0,$$  \(28\)

where \(\theta = (r_l - r_u)/(1 + r_d) > 0\) is the present value of savings by financing through loan sales rather than internal funds.

Before attempting to analyze equations (27) and (28), let us consider the characteristics of the probability density function, \(f(x, a)\), for a typical bank loan, as this will prove insightful in interpreting these optimality conditions. Assume that the bank loan is made to an otherwise all-equity-financed firm that invests its funds in assets (projects) with an uncertain return. If \(V\) is the value of this firm's assets when its loan with promised payment \(L\) becomes due, then at maturity the value of this bank loan will be

$$x = \min[L, V].$$  \(29\)

A reasonable assumption concerning the range of the distribution of \(V\) is that it is bounded below at zero. In addition, we assume that the bank's monitoring level, \(a\), affects the form of the firm's asset density function such that a lower level of \(a\) implies a "fatter" lower tail of the density function of \(V\). Figure 1 gives a plausible form for this probability density function of \(V\); \(g(V, a_1)\) is the density if the firm is monitored by the bank at level \(a_1\), while \(g(V, a_0)\) is the density if the firm is monitored at level \(a_0\), where \(a_1 > a_0\).

Given the density function for \(V\), the density function for the loan return, \(x\), is determined. The loan-return density when the bank monitors at level \(a_1\), \(f(x, a_1)\), is simply equal to \(g(V, a_1)\) for \(V < L\), with all the probability mass of \(g(V, a_1)\) for \(V \geq L\) "piled" together at point \(L\). Thus, the value of \(f(L, a_1)\) is a Dirac delta function spike with area equal to \(\text{prob}(V \geq L)\). \(f(x, a_0)\) over the range \([0, L]\) will bear a similar relationship to \(g(V, a_0)\).

Note in Figure 1 that, if the promised loan payment, \(L\), is not too large relative to the density of \(V\), then \(g(V, a_0) > g(V, a_1)\) and, hence, \(f(x, a_0) > f(x, a_1)\) for

![Figure 1](image-url)

**Figure 1.** Firm asset \((V)\) and loan return \((x)\) probability density functions. --- \((g(V, a_1))\), density of firm's assets; ——— \((f(x, a_1))\), density of loan return; \(L\), promised loan payment; \(a_1, a_0\), monitoring levels, where \(a_1 > a_0\).
all $V$ and $x$ less than $L$. In other words, if the promised loan payment is sufficiently in the lower tail of the firm's asset return, then, over the range zero to $L$, the density is a decreasing function of the bank's monitoring level; i.e., less monitoring makes the tail "fatter".\textsuperscript{14} The casual observation that banks rarely make commercial loans carrying exorbitant interest rates, e.g., twenty points above prime, lends support to the proposition that $L$ is typically in the lower tail of the firm's asset distribution, where $g_a(V, a) < 0$.

Now, assuming, as in Figure 1, that $f_a(x, a) < 0$ for all $x < L$, we see from condition (28) that the expression within brackets must be non-negative for all $x < L$ such that

$$\mu(x) = \theta f(x, a) - \lambda f_a(x, a) \geq 0, \quad \forall x < L.$$  

(30)

Therefore, $b(x) = 1$ for all $x < L$; i.e., the optimal loan sale contract gives the loan buyer the entire loan return whenever a loan default occurs. Hence, the bank will receive a return from the loan only when the loan does not default since only when $x = L$ will $f_a(L, a)$ be positive. Only in this case will $\mu(L) = 0$ so that $b(L) < 1$ and

$$\lambda = \theta \frac{f(L, a)}{f_a(L, a)}.$$

(31)

Thus, our assumptions on the loan distribution and preferences lead to a unique piecewise-linear optimal sharing rule that looks very similar to the loan buyer having a debt position and the bank an equity position in the loan.\textsuperscript{15} The contract is characterized by penalizing the bank if low loan outcomes occur and rewarding the bank if high loan outcomes (no default) occur. Giving the bank a disproportionate share of the risk allows the bank to reap a disproportionate share of the gains from monitoring, enabling a greater amount of the loan to be sold while maintaining monitoring-incentive efficiency.

There is evidence that actual non-recourse loan sales contracts follow this principle of giving the selling bank a disproportionate share of the loan's risk. Melvin [20, p. 41] cites the example of Bank One's sale of promised payments from a pool of credit card receivables in which the selling bank retained an equity position in the pool equal to twice the historical default level of the receivables. A similar contract was designed to sell a senior interest in a pool of adjustable-rate commercial mortgages.\textsuperscript{16} Coast Savings and Loan, a California thrift that originated the mortgages, retained a junior twenty percent interest in the pool. Another example is the practice of many money-center banks of selling short-

\textsuperscript{14} One can think of the function of the bank's monitoring to be that of limiting the risk of the borrowing firm's projects (assets). The bank, by reducing the "fatness" of the tail of the firm's asset distribution, is improving the expected return on its loan.

\textsuperscript{15} The contract is not exactly debt-equity division of the loan return. Note that, for small loan default, i.e., $x = L - \varepsilon$, where $\varepsilon$ is a small positive quantity, the loan buyers could receive a total return greater than their return when the loan did not default at all since $b(x) = 1$ for $x < L$ and $b(x) < 1$ for $x = L$. The framework of Holmstrom [16] can be used to derive contract optimality conditions under an alternative assumption that bank-loan buyer asset choice displays risk aversion.

term "strips" of longer term loans. Typically, the originating bank negotiates a lending commitment of between one and seven years with a borrowing firm. This loan is then financed by selling short-term obligations of between one and three months, called "strips", to a loan buyer. When the strip matures, the loan buyer is under no obligation to renew this short-term financing. While the buyer of the strip is exposed to default risk in the short run before the maturity date of the strip, the originating bank retains greater exposure to default in the longer run because of its commitment to refinance the loan. This arrangement would therefore preserve much of the bank's incentive to monitor.

It is straightforward to show that the equilibrium level of bank monitoring that results under the optimal loan sales contract will be less than the most economically efficient (first-best) level of monitoring. Note that the terms in square brackets in equation (27) are just the second-order condition regarding the bank's optimal monitoring choice, which is assumed to hold and therefore is negative. This implies that the loan buyer's expected benefit from greater monitoring is positive:

$$\int_{0}^{L} b(x) x \, dF_a(x, a) > 0. \quad (32)$$

Rearranging the bank's incentive-compatibility condition (23), it follows that

$$\int_{0}^{L} x \, dF_a(x, a) = c'(a) + \int_{0}^{L} b(x) x \, dF_a(x, a) > c'(a). \quad (33)$$

Therefore, in equilibrium, the expected marginal return on the loan from additional monitoring exceeds the marginal cost to greater monitoring.

Certain loans may not require any internal financing under the equilibrium bank-loan buyer contract, i.e., $I = 0$. In this case, the structure of the contract remains qualitatively the same, however, with a return going to the bank only if the loan return is high. Clearly, loans for which the benefits from monitoring are negligible or zero would be fully sold. For example, the polar case in which $f_a(x, a) = 0$ for all $(x, a)$ would imply an equilibrium level of monitoring of $a = 0$. No problem of moral hazard would exist, and the bank would optimally choose

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18 Merton [21] shows that the risk premium on a promised corporate payment rises as the time until payment is received increases. Note, however, that it may not be the case that the bank has made a firm commitment to the borrower to renew the loan. As with other types of loan commitments, these contracts typically contain a rather vague condition whereby the bank would not be obligated to reextend, such as "if there was a materially adverse change in the borrower's condition." In practice, banks seldom utilize this condition even if it appears to be in their best interests at the time. Gorton and Haubrich [13] argue that there may still exist an implicit firm commitment to renew the loan that is bound by the bank's reputation as a dependable provider of loan commitment services.

19 When $I = 0$, condition (26) implies that $\omega \leq 1 + r_L$. For $1 + r_L \leq \omega \leq 1 + r_l$, condition (25) indicates that the bank will again receive a share of the loan's return only when no default occurs. For $0 \leq \omega \leq 1 + r_s$, $b(x)$ will tend to be less than one for large values of $x$ and equal to one for smaller values of $x$. 
to finance the loan entirely through funding from loan sales.\footnote{While Greenbaum and Thakor’s \cite{Greenbaum} signalling model produces a distinct (and perhaps complementary) rationale for loan sales, they arrive at a result similar to that of this paper—namely, that better quality assets are funded by loan sales while poorer quality assets (cf. those needing more monitoring) are funded with deposits.} This (degenerate) case could be viewed as the bank performing solely an underwriting function. Of course, this makes sense since, if there were no benefits to monitoring a borrowing firm’s loan, the loan would be essentially a marketable asset.

\section*{B. Loan Sales with Recourse}

One simple contractual arrangement that could provide a first-best solution to the above problem (20) to (22) would be for the bank simply to guarantee the loan buyer a rate of return of $r_d$ regardless of the actual loan payoff. This contract, similar to the bank-depositor contract in Diamond \cite{Diamond}, would be feasible if the bank were able to maintain sufficient asset (loan) diversification such that the probability of the bank’s failure were negligible. Giving the loan buyer recourse to claims on other bank assets conceivably would allow the bank to sell the entire loan and still retain the incentive to monitor at the economically efficient level.

However, the Federal Reserve has sought to place restrictions on direct guarantees on loan sales.\footnote{See Federal Reserve regulations 12 CFR Part 204 Regulation D; Docket No. R-0571 and the instructions for filing Reports of Condition and Income, as well as the explanations of these regulations in Pavel \cite{Pavel}. While the Federal Reserve places restrictions on direct guarantees by banks of loan sales, they have not acted to restrict loan sales that are guaranteed by third-party insurance companies, even if the bank and insurance company negotiate an agreement that obligates the bank to reimburse ex post the insurance company for any payments it must make to the loan buyer. Under these circumstances, third-party insurance of loan sales appears to be the optimal arrangement from the bank’s and loan buyer’s points of view. However, it is reasonable to believe that the existence of this loophole will be short-lived.} With only a few exceptions, guarantees by banks to reimburse loan buyers for loan losses, even if a ceiling on the amount of the bank’s reimbursement were made, would lead the Fed to treat the bank’s proceeds from a loan sale as a “deposit” subject to inclusion in calculations of required capital and possibly subject to required reserves. Federal Reserve proposals do allow for loans to be sold with recourse in the following manner. If a bank agrees to guarantee a given \textit{percentage} of loan sales losses, say $l$, where $l$ is less than seventy-five percent, then this bank will be permitted to classify only a proportion $(1 - l)$ of the proceeds from the loan buyer as a loan sale. The other proportion, $l$, of the proceeds must be classified as a deposit, again subject to required capital. However, the Federal Reserve has stated that, as long as the percent losses guaranteed, $l$, is less than seventy-five percent, no required reserves need be held against the proceeds of the loan sale.

Under this arrangement, where the loan buyer’s payment is $b(x)x = bx + bl(L - x)$, one might ask what the bank’s optimal choice of $b$ and $l$ is. For this type of recourse loan, the bank’s problem is

$$\max_{b, l, a} \left[ \alpha(a) - b[\alpha(a) + l(L - \alpha(a))] - c(a) - (1 + r_I)I \right], \quad (34)$$
subject to

\[
\begin{align*}
\frac{b[\ddot{\xi}(a) + l(L - \ddot{\xi}(a))]}{1 + \frac{1 - l}{1 + r_d} + \frac{l}{1 + r_c}} + I &= 1, \\
[1 - b(1 - l)]\ddot{\xi}_a &= c'(a), \\
0 \leq l &\leq T = 0.75, \\
0 &\leq b \leq 1,
\end{align*}
\]  

(35)  

(36)  

(37)  

(38)

where \( \ddot{\xi}(a) = \int_0^l x \, dF(x, a) \), \( \ddot{\xi}_a = \int_0^l x \, dF_a(x, a) \), and \( r_c \) denotes the cost of bank funds from loan sales that are subject to capital constraints but not required reserves. \( r_c \) will be the cost of capital on the proportion \( l \) of loan-sale proceeds when the proportion of loan losses guaranteed is less than seventy-five percent. Therefore, \( r_c \) equals the expression for \( r_l \) but where required reserves, \( \rho \), have been set equal to zero, and hence \( r_c \leq r_l \).

Assuming that the incentive-compatibility constraint (36) is binding in equilibrium, it is clear from (37) and (38) that the equilibrium level of bank monitoring, \( a \), will be less than the economically efficient (first-best) level. Interpreting the Kuhn-Tucker conditions from (34) to (38), it is straightforward to show that possible optima exist only for two sets of \( (b, l) \) combinations: where constraints (36) and (37) bind and where constraints (36) and (38) bind. Therefore, banks will always choose a positive level of loss guarantees under these regulations. Depending on the magnitude of the parameters of the model, the bank will either sell the entire loan with \( l < T \) or choose to sell somewhat less than the entire loan with \( l = T \).

III. Loan Sales and Imperfect Competition in Deposit Markets

Money-center and large regional banks account for virtually all the recent growth in commercial and industrial loans sold in the U.S.\textsuperscript{22} The majority of these sales are portions of short-term loans or loan strips of investment-grade borrowers, implying that these assets are similar in quality to money-market instruments such as commercial paper. It could be argued that some of this loan selling substitutes for commercial-paper underwriting, an activity where commercial banks face restrictions.

Purchasers of these loans include a growing number of non-bank financial institutions and nonfinancial corporations. However, the bulk of these assets are bought by smaller domestic banks and foreign banks. One explanation for why other banks might wish to purchase these short-term loans is that they serve to enhance a bank’s liquidity, reducing the cost of providing transactions services such as check clearing and currency-to-deposit convertibility.\textsuperscript{23} A second expla-

\textsuperscript{22} The Board of Governors of the Federal Reserve System’s February 1986 Senior Loan Officer Opinion Survey gives information on the sixty largest U.S. banks’ commercial loan sales activities, including their loan sales volume and the principal purchasing institutions. The nine largest banks accounted for more than half of the $26 billion loan sales outstanding as of year-end 1985.

\textsuperscript{23} A transactions demand for liquid or marketable assets could be derived similarly to the inventory-theoretic transactions demand for money modeled by Miller and Orr [24].
nation for banks purchasing loans emerges by generalizing the model in Section 1 to consider imperfect competition in banks' deposit markets. The analysis is similar to that of Fama [9]. Banks with market power in deposit financing, but with relatively weak loan-origination opportunities, become candidates for loan purchases.

To illustrate this point in a simple manner, assume that, if an originating bank monitors a loan at level \( \alpha = \hat{\alpha} \), there is zero probability of default and the loan returns \( x(s, \hat{\alpha}) = 1 + r_n \). Monitoring at any level \( \alpha < \hat{\alpha} \) results in a loan default, with the loan returning \( 1 + r_n - z \), where \( z > \hat{\alpha}c \) is constant for all states. Let \( \bar{b} \) equal the proportion of financing obtained through selling a share of the loan's return, where \( \bar{b} \) is assumed sufficiently small to maintain the bank's incentive to monitor. In addition, let \( r_m \) denote the yield on money-market instruments and competitively priced deposits, and, as before, let \( r_I \) equal the cost of internal financing when bank deposits are competitively priced.

Figure 2 depicts an equilibrium that corresponds to the loan sales equilibrium previously analyzed in Section I, Subsection B. \( NN' \) denotes the marginal-revenue curve for loans, while \( NAA' \) denotes the bank's marginal-revenue curve for all assets, assuming that the bank can always purchase marketable assets bearing a return of \( r_m \) instead of originating loans past the point \( r_n - c\hat{\alpha} < r_m \).

The marginal cost of funds is given by \( DD' \) when the bank does not sell loans and \( DSS' \) when the bank does. Note that, for this case, the point \( S \) is to the left of point \( A \), indicating that the marginal cost of funds rises above the level \( r_m \) prior to the marginal-revenue curve, \( NAA' \), and implying that the bank will optimally begin selling loans at point \( S \). Therefore, the case depicted in Figure 2 describes a bank that has relatively greater loan-origination opportunities than core-deposit funding opportunities.

In contrast, Figure 3 depicts an equilibrium in which a bank has relatively less loan-origination than deposit-funding opportunities. Here, the marginal-cost-of-funding curve \( DSS' \) reaches the level \( r_m \) at point \( S \), which is to the right of where

![Figure 2. Loan-selling bank. \( NN' \), marginal revenue from loan origination = \( r_n(N) - c\hat{\alpha} \); \( NAA' \), marginal revenue for all assets; \( DD' \), marginal cost of funds without loan sales; \( DSS' \), marginal cost of funds with loan sales.](image)
the marginal-revenue curve $NAA'$ reaches $r_m$ at the point $A$. This bank will optimally purchase marketable assets, which pay return $r_m$, equal in value to the length $AS$. Some of these marketable assets could take the form of loan shares sold by a bank with a situation of that of Figure 2. Therefore, the analysis in Figures 2 and 3 provides testable implications regarding which banks would choose to sell versus buy loans or hold marketable securities. Furthermore, the model predicts that a decline in inexpensive deposit-funding opportunities would tend to result in an expansion of loan sales.

These implications are roughly consistent with the stylized facts of loan sales. Large money-center banks, which originate the lion’s share of loans sold, generally acquire funding in wholesale markets, through issuing large certificates of deposit and other purchased funds, paying competitive rates. Smaller banks have a much greater proportion of their liabilities in small time and savings deposits—approximately sixty-nine percent for banks with assets of less than $100$ million versus thirty-six percent for banks with assets exceeding $100$ million. Peltzman [28, p. 562] states, “If small banks have had a cost advantage anywhere, it has been in securing deposits. . .[time] deposit cost differences between small and large banks running on the order of 200 to 400 basis points.”

While larger banks are at a relative disadvantage in terms of deposit funding, one would expect that their loan-origination opportunities might exceed those of many smaller banks. Because of their location in large financial centers throughout the world, money-center banks would appear to have access to many potential borrowers. Small banks constrained to local lending markets might also have the amount of their lending opportunities limited because of an inability to achieve diversification, which purchasing loans can help to remedy. A recent empirical

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$^{24}$ See Waldrop [33] for a comparison of large and small banks' liability compositions. Note that Banker's Trust is one of the first and largest banks to expand its loan-sales activities. In 1979, it sold eighty branches in its retail network having combined deposits of $934$ million, receiving a $85$ million sales premium on these deposits.
study by Pavel and Phillis [27] finds that a bank's comparative advantage in originating and servicing loans has the largest impact in determining the amount of loans that it will sell. The size of a bank's assets, its ability to diversify, and whether its capital constraint is binding are also significant factors in determining the amount of its loan sales. James [17] also finds that banks with a large volume of off-balance-sheet activities tend to have high leverage.

Finally, the growth in the aggregate volume of loan sales has roughly coincided with a general decline in banks' market power in deposit financing. The 1970's and early 1980's saw a disintegration of the monopolistic price-fixing effects of Regulation Q as unregulated intermediaries such as money-market mutual funds provided competitive returns to small investors. Securitization of mortgage loans showed steady growth over this period, while commercial, credit card, and auto loan sales totaling a relatively negligible amount in 1983 are estimated by Salem [31] to stand at roughly $35 billion in 1986, up approximately one hundred percent from the previous year.

IV. Conclusion

This paper demonstrated that banks faced with significant competition for deposit financing, as well as regulatory constraints in the form of required capital and/or reserves, cannot profit by simply holding money-market assets but must provide other services, such as information gathering and monitoring activities related to making loans. Loan sales can reduce the cost of funding these loans. However, we showed that other banks with substantial market power in deposit financing, but with limited loan-origination opportunities, may choose to hold marketable assets. These assets can take the form of loan shares purchased from those banks facing competitive financing.

A bank's ability to sell loans depends on loan buyers' perception of the bank's incentive to monitor those loans. By designing the loan sales contract in a way that gives the bank a disproportionate share of the gains to monitoring, it was shown that a greater share of the loan can be sold and, hence, a greater level of bank profits can be attained.

There is another issue that this paper has touched upon but not adequately treated. Determining what effect greater loan sales has on overall bank risk would be a productive area of research. Because the optimal loan-sales contract attempts to give the bank a disproportionate share of the gains to monitoring, the bank will generally be assuming a disproportionate share of a given loan's risk. Thus, it might appear that loan sales would increase the volatility of the bank's asset portfolio. However, for a given bank capital structure, the benefits of asset diversification, deriving from a greater number of loans originated when loan selling occurs, might outweigh the higher risk incurred on each individual loan. It may be unwise for regulators to unconditionally discourage loan sales.

Appendix

Below is a derivation of the bank's objective function when banks maximize the after-tax rate of return to shareholders and deposit insurance is fairly priced.
Using the notation in the text, the bank’s end-of-period after-tax asset value, when state \( s \) occurs, is

\[
\{ \sum_{i=1}^{N} (x_i(s, a_i) - 1 - ca_i) - r_d D \} (1 - t) + N + \rho D + r_d D. \tag{A1}
\]

Letting \( \phi \) be the premium charged for deposit insurance, the payoff to equityholders when the bank is solvent is

\[
\{ \sum_{i=1}^{N} (x_i(s, a_i) - 1 - ca_i) - r_d D \} (1 - t) + E - \phi D = W(s) - \phi D. \tag{A2}
\]

Of course, equityholders receive nothing when bankruptcy occurs. The end-of-period payoff to the deposit insurer is \( \phi D \) when the bank is solvent and \( W(s) \) when the bank is insolvent. The insurer receiving \( W(s) \) implicitly assumes that the value of the bank’s tax shield is preserved when the bank fails. This is not unrealistic since Kane [18, p. 38] points out that, when a failed bank is merged with an acquiring bank, the failed bank’s losses can be used to reduce the acquiring bank’s tax liability.

It is assumed that the deposit insurer’s regulatory authority enables it to costlessly audit the bank’s risk just after the start of the period. Let \( S_0 \) denote the set of solvency states and \( S_1 \) the set of insolvency states. Also let \( \{ p(s) \} \) denote the set of primitive security prices that the deposit insurer uses to value its end-of-period cash flow. Then a fair premium, \( \phi \), is such that

\[
\phi D \int_{S_0} p(s) \, ds + \int_{S_1} p(s) W(s) \, ds = 0. \tag{A3}
\]

Substituting this value for \( \phi \) in (A2) and taking the shareholders’ present value of (A2) over all solvency states, \( S_0 \), one obtains

\[
\int_{S_0} W(s) p^*(s) \, ds + \int_{S_1} p^*(s) \frac{W(s) \, ds}{p(s) \, ds} \int_{S_0} p^*(s) \, ds. \tag{A4}
\]

Finally, the assumption is made that \( p(s) \) is a constant proportion of \( p^*(s) \) for all \( s \). For example, if the insuring agency’s valuation of contingent claims reflected that of depositors (debtholders) with state-contingent prices \( \{ p^d(s) \} \), then, as shown by Litzenberger and Van Horne [19, p. 739], the quantity \( p^d(s)/p^*(s) \) would be constant across states if the personal tax bracket of the “marginal” investor indifferent between holding debt and equity were constant across states. This assumption is also made in DeAngelo and Masulis [5].

Expression (A4) can then be written as

\[
\left\{ \sum_{i=1}^{N} \left[ (1 + r_e) \int_{0}^{1} p^*(s)x_i(s, a_i) \, ds - 1 - ca_i \right] - r_d D \right\} \frac{(1 - t)}{(1 + r_e)} + \frac{E}{1 + r_e}. \tag{A5}
\]

If the bank is assumed to maximize the difference between the present value of equityholders’ payment (A4) and the amount of equity that must initially be raised, \( E \), then, by subtracting \( E \) from (A4) and multiplying by \( (1 + r_e) \), one obtains equation (2) in the text.
REFERENCES


